Exam. Code : 103201
Subject Code : 1026

B.A./B.Sc. 1st Semester

MATHEMATICS

Paper—II (Calculus & Trigonometry)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt *five* questions in all, selecting at least *one* question from each section. The **fifth** question may be attempted from any section.

SECTION—A

1. (a) Prove that between any two distinct real numbers, there is always a rational number, and therefore, infinitely many rational numbers.

   (b) Show that the function $f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$, where $x \neq 0$ and $f(0) = 0$ is discontinuous at $x = 0$. 5,5

2. (a) Find least upper bound and greatest lower bound, if exists, for the set $A = \left\{\frac{2-x}{1-x}, x > 0,\ x \neq 1\right\}$.

   (b) Prove that the function $f(x) = \sin\left(\frac{1}{x}\right); x \in (0,1]$ is not uniformly continuous. 5,5

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SECTION—B

3. (a) Prove that $\tanh^{-1}x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right), -1 < x < 1$, and then find its derivative w.r.t. x.

(b) If $y = \sin(m \sin^{-1}x)$, find $y_n(0)$. 5,5

4. (a) Evaluate : $\lim_{x \to 0^+} x^m (\log x)^n$, m and n $\in$ N.

(b) State and prove Taylor's theorem with Lagrange's Form of Remainder. 5,5

SECTION—C

5. (a) Prove that the q distinct values of $(\cos\theta + i\sin\theta)^{\frac{p}{q}}$ form a G.P. and their sum is zero.

(b) Show that each primitive 8^{th} root of unity satisfies the equation $z^4 + 1 = 0$. 5,5

6. (a) If $\log[\sin(\theta + i\phi)] = \alpha + i\beta$, show that :

$$\cosh 2\phi - \cos 2\theta = 2.e^{2\alpha}$$

(b) Show that the roots of the equation :

$(1 + x)^n = (1 - x)^n$, (n being +ve integer)

are $i\tan\left(\frac{b\pi}{n}\right)$, b = 0,1,2, ..., n–1. 5,5

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SECTION—D

7. (a) If $i^{i^{\cdots\infty}} = A + iB$ and only principal values are considered, prove that :

(i) $\tan\frac{\pi A}{2} = \frac{B}{A}$;

(ii) $A^2 + B^2 = e^{-\pi B}$.

(b) Prove that :

$$\cos^8\theta = \frac{1}{2^7}[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35]$$

5,5

8. (a) Show that :

$$\sin x + n\sin(x+y) + \frac{n(n-1)}{1.2}\sin(x+2y) + \text{........} + \text{........ upto } (n+1) \text{ terms}$$

$$= \left(2\cos\frac{y}{2}\right)^n \sin\left(x + \frac{ny}{2}\right).$$

(b) Use Gregory series to prove that :

$$\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{8}\right) - \frac{1}{3}\left(\frac{1}{2^3}+\frac{1}{5^3}+\frac{1}{8^3}\right) + \frac{1}{5}\left(\frac{1}{2^5}+\frac{1}{5^5}+\frac{1}{8^5}\right) + \text{.......} = \frac{\pi}{4}.$$

5,5

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